GENERALIZED SPEED-TO-FLY THEORY

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INTRODUCTION

Classic (MacCready) speed-to-fly theory covers only a special case of a simple climb and cruise cross-country flight. It assumes that the sources of lift are stationary in respect to the air mass, i.e. that they drift downwind at exactly the same speed as that of the wind.

This paper introduces a generalized speed-to-fly theory, which additionally deals with the updrafts that move more slowly than the prevailing wind, or do not move at all. A new variable, named the coefficient of updraft drift (C_{ud}), is used to describe the horizontal movement of lift sources. Methods for calculating the correct speed-to-fly and the average cross-country speed for any combination of wind and C_{ud} are given.

Generalized speed-to-fly theory does not deal with dolphin flight, either static nor dynamic. It only enhances the classic MacCready theory to include all kinds of lift.

CLASSIC THEORY

For the sake of comparison, we will make a short review of the classic speed-to-fly theory. According to it, the optimum cruising speed in still air is determined by only two factors: the rate of climb and the speed-sink polar of a glider. For a given polar, the speed-to-fly depends solely on the climb rate in a thermal, or any other updraft.

The optimum cruising speed curve can be determined graphically, by drawing tangents to the polar from different rate of climb origins (Figure 1). The speed rings and the electronic speed-command instruments are designed using this curve.

The average cross-country speed in no wind condition (V_{xco}) can be obtained from the same diagram, or calculated from the following relation:

$$V_{xco} = \frac{V_g W_c}{W_g + W_c} \qquad (1)$$

where W_c is the rate of climb, V_g is the cruising airspeed, and W_g is the glider sink rate in cruise (at V_g).

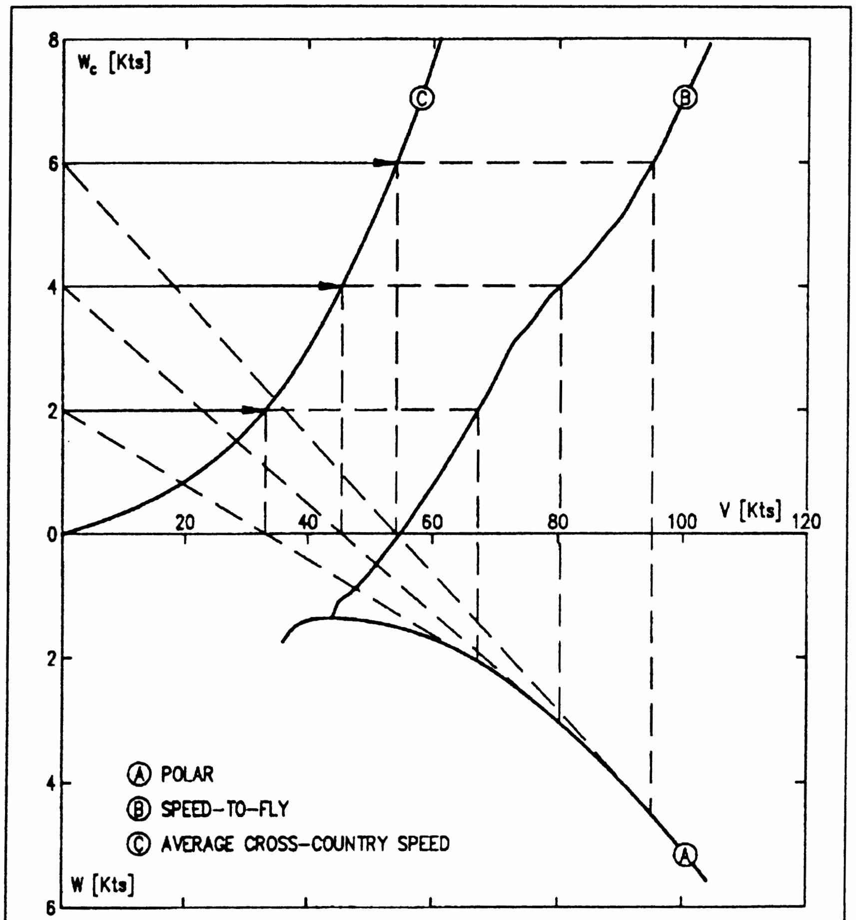

FIGURE 1. Graphic presentation of the classic theory.

If the air between the updrafts moves vertically, the polar curve will be shifted up or down, and the shift will be equal to the vertical speed of the air mass W_W. (W_W is assumed positive when the air moves upward.) In this case, equation (1) should be adjusted by substituting W_g with $W_g - W_W$. The optimum cruising speed can be obtained from the still air speed-to-fly curve, by subtracting W_W from the rate of climb W_c.

When a horizontal wind is present, the classic theory suggests that the optimum cruising speed stays the same as in no wind conditions. The average cross-country speed is affected by the wind, and it's corrected value can be easily obtained from a wind triangle. This is done by computing the vector sum of the average speed for no wind V_{xco} and the wind speed V_W (figure 2). The corresponding equation is:

$$V_{xco} = \sqrt{V^2_{xco} - V^2_W \sin^2 ß} - V_W \cos ß \qquad (2)$$

where ß is the wind angle. (ß = 0° in head wind, ß = 90° in cross wind and ß =180° in tail wind conditions).

GENERALIZED THEORY

The precedinq consideration is valid under the assumption that updrafts drift completely with the wind. However, this assumption is correct only in case of thermal updrafts in calm and light wind conditions. In stronger winds, and particularly when there is a strong wind shear, thermals generally lag behind the prevailing wind. As a thermal rises from a level of weaker wind into a level of stronger wind, it tends to keep it's original momentum, thus moving more slowly than the surrounding air.

Ridge lift and lee waves feature updrafts that are stationary with respect to the ground. In this case, the assumption made by the classic theory is completely wrong.

As a first step towards a generalized speed-to-fly theory, we will define the coefficient of updraft drift C_{ud} as the ratio between the speed of updraft horizontal movement V_{ud} and the wind speed V_W:

$$C_{ud} = \frac{V_{ud}}{V_W} \qquad (3)$$

$C_{ud} = 1$ means that updrafts move with same speed as the wind, and $C_{ud} = 0$ describes the stationary sources of lift. The only assumption made here, is that updrafts move in the same direction as the wind. This is generally true, except for thermals when there is a significant change in wind direction with altitude. In that case, the assumption is not strictly met, but it still provides a better approximation than the classic theory.

Next, we will define three reference systems to be used for measuring horizontal speeds and angles:

1. Ground reference system is f ixed to a point on the ground.
2. Wind reference system moves with the air mass, at the speed that equals the wind speed V_W.
3. Updraft reference system moves horizontally with the updraft. Viewed from the ground reference system, the speed at which the updraft system moves is V_{ud} (where $V_{ud} = C_{ud} V_W$). Measured in the wind system, the speed of the updraft system is $V_W - V_{ud}$, o r $V_W (1 - C_{ud})$.

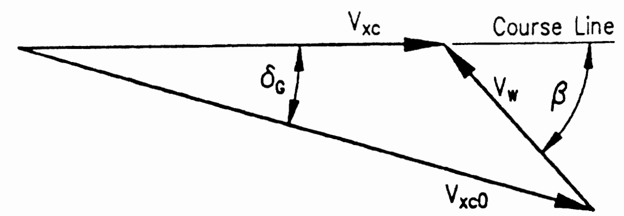

FIGURE 2. Average X-country speed wind triangle (classic theory).

HEAD / TAIL WIND

We will begin our analysis with a simple case involving only head or tail wind. Consider a glider cruising into a head wind, toward a stationary wave ($C_{ud}=0$). The speed at which the glider is approaching the updraft equals glider airspeed minus wind speed ($V_g - V_W$). That has the same effect as if the original polar (curve 'A' in Figure 3) was shifted toward lower speeds by the amount equal to the wind speed (curve 'B'). Therefore, the correct speed-to-fly, can be found by drawing tangents to polar 'B', instead of polar 'A'. Note that polars 'A' and 'B' are actually drawn in wind and updraft reference systems, respectively.

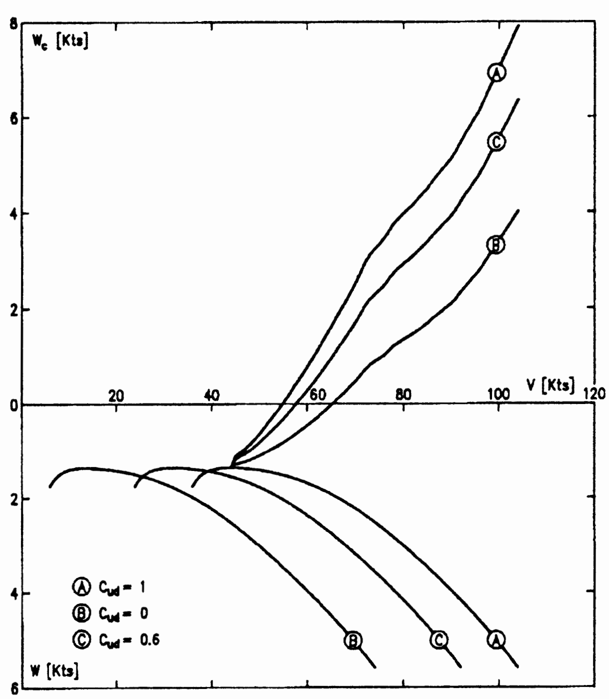

FIGURE 3. Polar shift in a 30 kt head wind.

In a more general case, the updraft horizontal movement can be anywhere between zero and the full wind speed ($0 \leq C_{ud} \leq 1$). To obtain the optimum speed-to-fly, we will again measure the glider horizontal speed in the updraft reference system. The amount of polar shift ΔV, is now determined by the difference between the wind speed, and the updraft horizontal movement:

$$\Delta V = V_W - V_{ud} = V_W\,(1 - C_{ud}) \qquad (4)$$

The resulting polar 'C' will lie somewhere between polars 'A' and 'B'. The optimum speed-to-fly curve will be different for different values of ΔV, and it will change with both the wind speed and the coefficient of updraft drift. Relation (4) shows that when C_{ud} equals one, there is no polar shift, regardless of the wind speed.

A similar analysis can be performed in case of a tail wind. The only difference is that the polar offset will be towards higher speeds.

CROSS WIND

Another special case is a 90° cross wind. Let us again start with stationary updrafts ($C_{ud}=0$). While cruising in these conditions, a glider must crab Into the wind to stay on course. Since the sources of lift are stationary, the updraft reference system coincides with the ground system, and the crab angles in both systems are identical ($\delta_U = \delta_G$). The crab angle can be found from a vector triangle involving the cruising speed V_g and the wind speed V_W (see figure 4):

$$\sin \delta_U = \sin \delta_G = \frac{V_W}{V_g} \qquad (5)$$

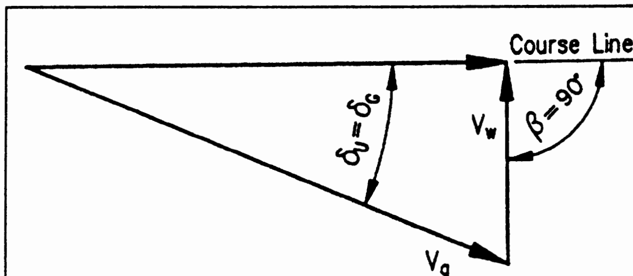

FIGURE 4. Crab angle in cross wind for $C_{ud} = 1$.

Viewed from the updraft reference system, the polar shift now depends not only on the wind speed, but also on the cruising speed of a glider:

$$\Delta V = V_g\,(1 - \cos \delta_U) \qquad (6)$$

Equations (5) and (6) show that the polar will be shifted more in the lower speed region, and less at the high speed end. This means that the shape of the polar, as well as it's position will be changed (polar 'B' in figure 5). The optimum speed-to-fly curve can again be obtained by drawing tangents to the modified polar.

For $0 \leq C_{ud} \leq 1$, the polar shift can be calculated in the similar fashion. However, the crab angles δ_U and δ_G

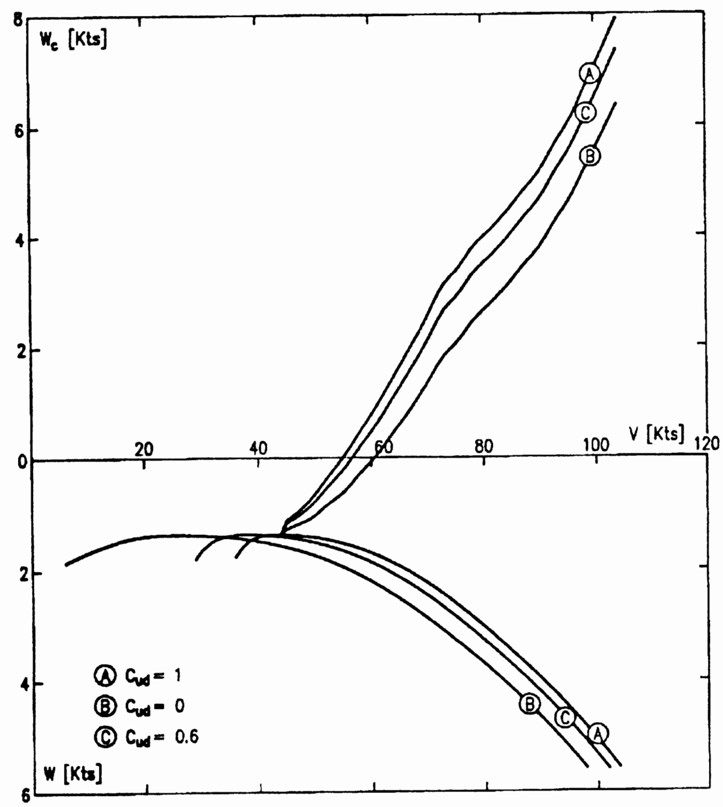

FIGURE 5. Polar shift in a 35 kt cross wind.

cannot be assumed equal any more. In first approximation, δ_U is given by:

$$\sin \delta_U = \frac{V_W (1 - C_{ud})}{V_g} \qquad (7)$$

The resulting polar 'C' in figure 5, lies between polars 'A' and 'B'. Relation (7) also shows that when $C_{ud} = 1$, the crab angle measured in the updraft reference system δ_U is zero, regardless of the wind speed. In that case, the polar offset is also zero.

The exact solution for δ_U is much more complex than the one suggested by (7). It also involves the climb rate W_c and the sink rate in cruise W_g. However, relation (7) is valid in both boundary conditions ($C_{ud} = 0$ and $C_{ud} = 1$), and for a C_{ud} that is somewhere in between, the resulting speed-to-fly error is not significant.

GENERAL WIND CONDITIONS

So far, we have shown that the problem of finding the correct speed-to-fly in conditions where updrafts move slower than the wind, can be reduced to finding the corresponding polar shift. Once we know how to obtain the polar shift in head/tail and cross wind conditions, we can resolve any wind condition by separately considering the head/tail and cross wind components of the wind vector (figure 6). The resulting polar shift will be the sum of the shifts caused by both components.

AVERAGE CROSS - COUNTRY SPEED

Second part of the generalized speed-to-fly theory deals with the average cross-country speed in windy conditions for $0 \le C_{ud} \le 1$. We will base our consideration on one cruise-climb cycle, which starts and ends at the same altitude. Figure 7 shows a top view of one such cycle in a ground reference system.

The cruise part of the cycle starts at point 'A', and ends at point 'C'. Note, that if there was no wind, the glider would reach point 'B' at the end of the cruise. The climb starts at point 'C' and ends at point 'D', where the whole cycle also ends. The average cross-country speed V_{xc} can be expressed as the total distance covered over duration of the cycle:

$$V_{xc} = \frac{L}{T} \qquad (9)$$

where

$$L = \overline{AD} \qquad (10)$$

The duration of a cruise-climb cycle can be divided into glide (cruise) duration T_g, and climb duration T_c:

$$T = T_g + T_c \qquad (11)$$

T_g and T_c can be expressed as:

$$T_g = \frac{H}{W_g}; \quad T_c = \frac{H}{W_c} \qquad (12)$$

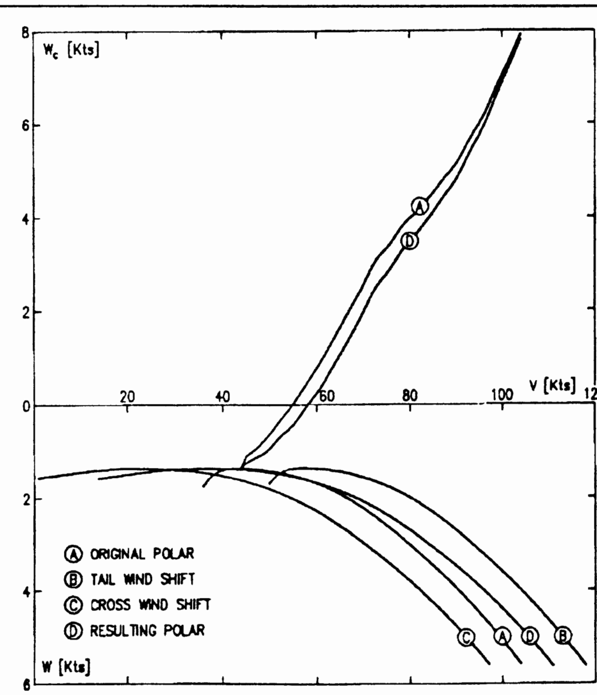

FIGURE 6. Polar shift in a 40 kt wind (ß = 110°; C_{ud} = 0).

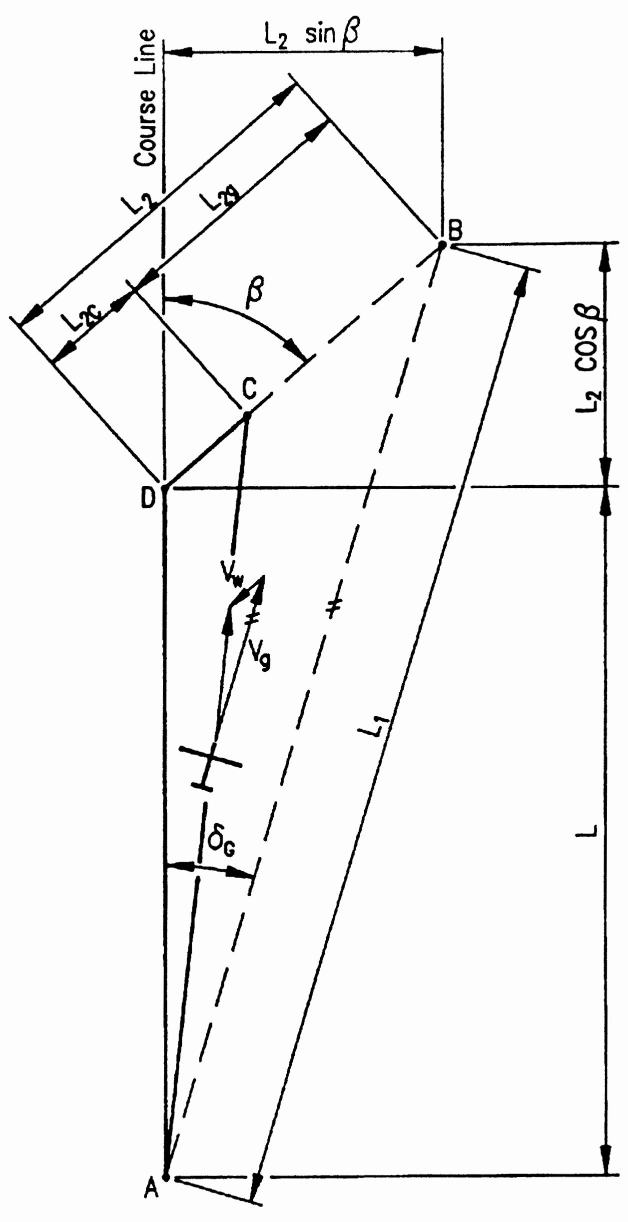

FIGURE 7. Cruise-climb cycle in ground reference system.

where H is the altitude lost in glide and then regained in climb, W_g is glider sink speed during glide, and W_c is the attained rate of climb.

Considering the trigonometric relations in figure 7, we can write:

$$L = \sqrt{L_1^2 - L_2^2 \sin^2 ß} - L_2 \cos ß \qquad (13)$$

Total drift during one cycle L_2 can be divided into glide portion L_{2g} and climb portion L_{2c}:

$$L_2 = L_{2g} + L_{2c} \qquad (14)$$

where $$L_{2g} = V_W T_g = \frac{V_W H}{W_g} \qquad (15)$$

$$L_{2c} = C_{ud} V_W T_c = \frac{C_{ud} V_W H}{W_c} \qquad (16)$$

C_{ud} is, of course, the coefficient of updraft drift. From (14), (15) and (16), L_2 can be expressed as:

$$L_2 = V_W H \left[\frac{1}{W_g} + \frac{C_{ud}}{W_c} \right] \qquad (17)$$

L_1 is the distance traveled in the wind reference system. It is given by:

$$L_1 = V_g T_g = \frac{V_g H}{W_g} \qquad (18)$$

After substituting (17) and (18) into (13), we obtain:

$$L = H \left[\sqrt{\frac{V_g^2}{W_g^2} - C^2 \sin^2 ß} - C \cos ß \right] \qquad (19)$$

where C is a substitute for:

$$C = V_W \left[\frac{1}{W_g} + \frac{C_{ud}}{W_c} \right] \qquad (20)$$

Finally, from (9), (11), (12) and (19), we can write the complete equation of the average cross-country speed:

$$V_{xc} = \frac{W_c W_g \left[\sqrt{\frac{V_g^2}{W_g^2} - C^2 \sin^2 ß} - C \cos ß \right]}{W_c + W_g} \qquad (21)$$

If we compare (20) and (21) with the average cross-country speed equations given by the classic theory (1) and (2), we can see that the relation has become much more complex. We can no more use the graphic method to compute the average cross-country speed. Also it is not possible to produce a simple table, or an analog calculator that will give us either the speed-to-fly or the average cross-country speed in general conditions. The only solution lies in use of a computer.

Figure 8 shows the average cross-country speed curves for a Discus in a 25 knot wind.

SPECIAL CASES

Generalized speed-to-fly theory doesn't contradict the classic theory; it only covers a more general set of conditions. This can be proved by considering a few special cases.

Case a.) $C_{ud} = 1$

In this case, relation (21) can be reduced to:

$$V_{xc} = \sqrt{\frac{(V_g + W_c)^2}{(W_c + W_g)^2} - v_W^2 \sin^2 ß} - V_W \cos ß \qquad (22)$$

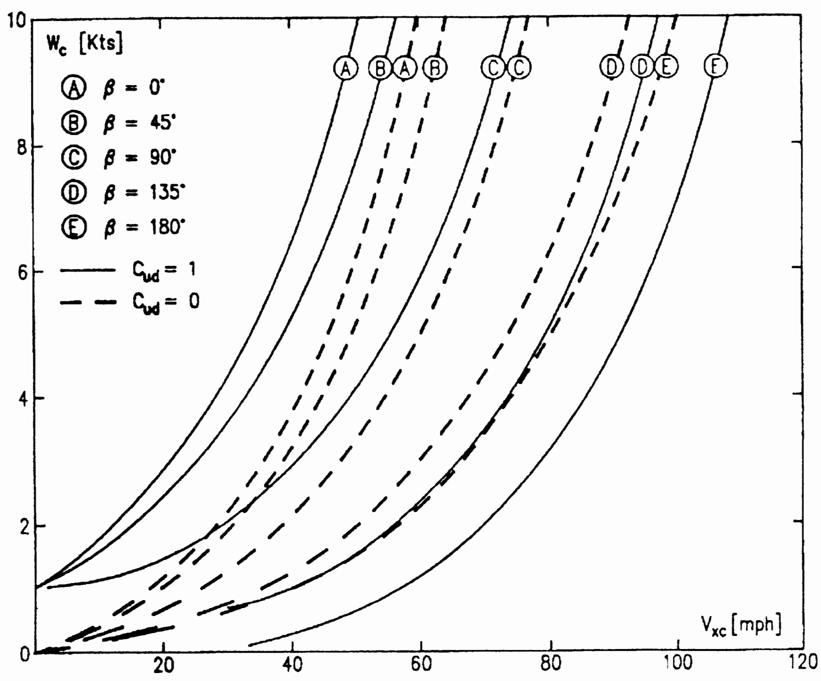

FIGURE 8. Average x-country speed in a 25 kt wind, Discus, W/S = 7 lb/ft², h = 3000 ft.

Substituting relation (1) into (22), we obtain

$$V_{xc} = \sqrt{V^2_{xco} - V^2_W \sin^2ß - V_W \cos ß} \qquad (23)$$

This is equivalent to the equation (2), showing that the generalized theory reduces to the classic theory whenever the coefficient of updraft drift equals one.

Case b.) Head/Tail Wind

In this case, we can substitute sin ß = 0 and cos ß = ± 1 into relation (21). After doing that, and taking into account (1), we obtain:

$$V_{xc} = V_{xco} \pm V_W \frac{W_c + C_{ud} W_g}{W_c + W_g} \qquad (24)$$

Again, if we assume C_{ud} = 1, as in the classic theory, we will get:

$$V_{xc} = V_{xco} \pm V_W \qquad (25)$$

PRACTICAL IMPLICATIONS

The most important shortcoming of the classic MacCready theory is inability to deal with wave and ridge cross-country flights. Since these sources of lift are stationary (C_{ud} = 0), the speed-to-fly suggested by the speed ring is usually way too low when flying into the wind, and much higher than optimum when flying downwind. The situation is further aggravated by the fact that both ridge lift and lee waves occur in moderate and strong winds.

A common practice among cross-country pilots is to carry more ballast and fly faster into the wind, and to carry less ballast and fly more slowly downwind. This can be both right or wrong, depending on weather the updrafts are stationary (as ridge lift and lee waves), or they more or less drift with the wind (as thermals do).

Generalized speed-to-fly theory provides a method of determining the optimum cruising speed in any conditions. However, there is no simple way of implementing it in practice, other than using a more sophisticated (and more expensive) electronic speed-to-fly instrument. Until the time such a device becomes available, we can use some more refined recommendations regarding flying in windy conditions:

1. In thermal conditions without a significant wind shear, we can assume that thermals drift completely with the wind. Here, we can use the same speed-to-fly as in no wind.
2. In thermal conditions with a pronounced wind shear, thermals lag somewhat behind the wind, and the coefficient of updraft drift will usually be between 0.5 and 0.9. Fair results can be achieved by setting the speed ring a little higher when flying into the wind, and a little lower when heading downwind. The amount of correction should depend on a given situation.
3. When using ridge or wave lift, we should fly considerably faster upwind, and more slowly down wind, than indicated by the speed ring. Again, how much faster or slower, depends on the wind speed, wind angle, climb rate, glider performance and wing loading.

A general advice is to make a good ground preparation. It is helpful to examine in advance the effects of the expected wind - C_{ud} combinations on the speed-to-fly, the average cross-country speed, and the optimum wing loading. (CuSoft Research Inc. has developed an interactive computer program that can, among other things, perform all these tasks. The program is called "Polar Explorer", and it will become available to the public by the end of 1991. The program runs on a PC compatible computer.)

MIXED CONDITIONS

In meteo conditions featuring both thermals and ridge or wave lift, estimating the correct speed-to-fly is

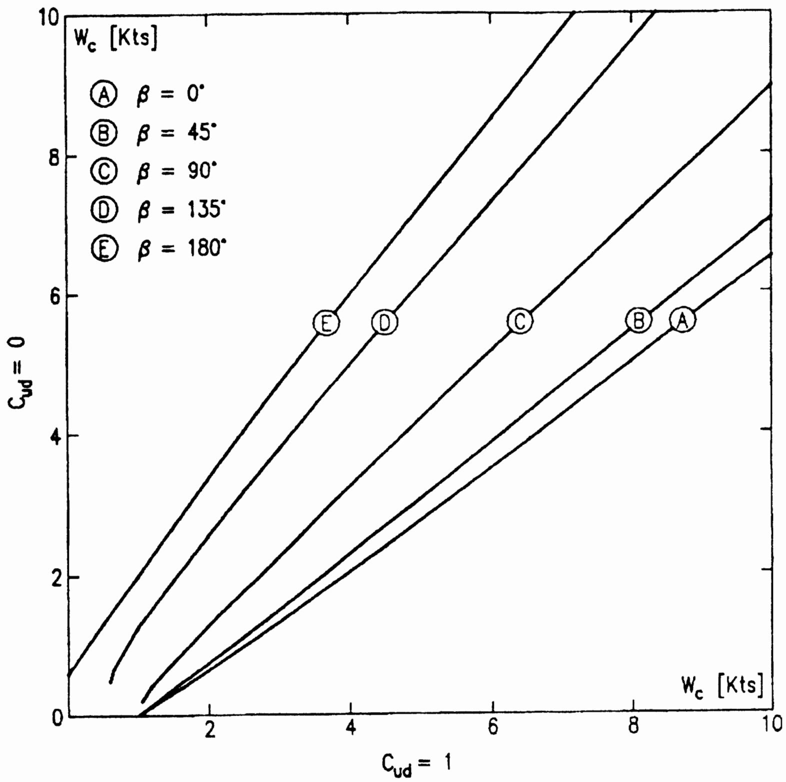

FIGURE 9. Break-even curves for 25 knot wind, Discus, W/S = 7.63 lb/ft, h = 3000 ft.

only a part of the problem. Maybe even more important, is to choose whether to climb in fixed or drifting sources of lift. The choice is simple if we can expect the same climb rate in either of them. Then, it is better to climb in thermals when going downwind, and in wave or ridge lift when flying into a head wind, or even a cross wind.

Now, consider flying a Discus with 7.63 lb/ft wing loading at 3000 feet, into a 25 knot head wind, and having to choose between a 8 knot thermal ($C_{ud} = 1$), and a 5 knot stationary wave ($C_{ud} = 0$). It is obvious that a climb in the thermal would take less time. However, it is difficult to tell whether the time saved by cllmbing in 8 knots rather than in 5 knots, would make up for the distance lost by being pushed back by the wind.

Generalized speed-to-fly theory has the answer to this kind of problem. Using relation (21), we can compute the average cross-country speeds for both sources of lift, and then decide which one is better. In head/tail wind conditions, it is enough to determine the optimum cruising speeds. In that case, the highest average cross-country speed will be achieved with the updraft for which the optimum cruising speed is the highest.

As you might have guessed, the optimum cruising speed in our example is 95.35 knots (equivalent airspeed) for both the 8 knot thermal ($C_{ud} = 1$), and the 5 knot wave ($C_{ud} = 0$). The corresponding average cross-country speeds are also identical at 48 mph.

If we plot all the points for which drifting and stationary updrafts produce the same average cross-country speed, we will obtain a break-even curve for a given set of conditions (see figure 9). Using a break-even curve is simple: if the point lies below the curve, use the drifting source of lift; if it lies above, use the stationary one.

CONCLUSION

Generalized speed-to-fly theory overcomes certaln restrictions imposed by the classic MacCready theory. Most notably, it enables proper treating of the cross-country flight tactics using semi-drifting and stationary updrafts. As the previous examples show, the errors made by applying the classic theory to these cases can be large, even in moderate wind conditions.

The main disadvantage of the new theory is its complexity. It does not enable us to use a simple device, such as the speed ring, to determine the optimum cruising speed in any conditions. A suitable electronic speed-to-fly indicator could be produced with today's technology, and the main problem might not lie in the design of such a device, but rather in it's use. Compared to the classic speed command instruments, this one would additionally require data concerning wind speed, wind angle and C_{ud}, to be able to function correctly. However, a thorough understanding of the new theory, may prove to be of more help to a cross-country pilot, than any new and more complex gadget.

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